Grade 7/8 Math Circles

Week of 10$^{\text{th}}$ October

Solids & Platonic Solids Solutions

Exercise Solutions

1. For the first shape, a square, we have that
   - Vertices: 4
   - Edges: 4
   - Faces: 1

   For the second, a trapezoid, we have that
   - Vertices: 4
   - Edges: 4
   - Faces: 1

   For the third, a hexagon, we have that
   - Vertices: 6
   - Edges: 6
   - Faces: 6
2. Starting from the left, we have (pentagon) convex, (cube) convex, (square based prism) convex, and (pentagram) concave.
3. We have, starting from top left, (pentagon) regular, (hexagon) regular, (isosceles triangle) irregular, (square) regular, irregular polygon, irregular polygon.
4. Starting from the left, we have regular, regular, irregular, and irregular. Since we know the faces of a regular polyhedron is a regular polygon, we notice that the first two are made from equilateral triangles and pentagons, respectively, which are regular polygons, and the last two are made from irregular polygons.
5. Starting from the top left, we have that the two squares are similar, with a scaling factor of $\frac{5}{7}$. The two trapezoids are *not* similar. The two triangles are congruent (note that to acquire the second triangle, just rotate the first 180$^{\text{o}}$. The final one is similar, with a scaling factor of $\frac{1}{2}$.

6. Starting to the left, we have
   (a) Equilateral triangle
      - Vertices: 4
      - Edges: 6
      - Faces: 4

   (b) Square
      - Vertices: 8
      - Edges: 12
      - Faces: 6

   (c) Equilateral triangle
      - Vertices: 6
      - Edges: 12
      - Faces: 8

   (d) Pentagon
      - Vertices: 20
      - Edges: 30
      - Faces: 12

   (e) Equilateral triangle
      - Vertices: 12
      - Edges: 30
      - Faces: 20

7. In the same order as the photo above, we have that
   - Tetrahedron: $\{3, 3\}$
   - Cube: $\{4, 3\}$
   - Octahedron: $\{3, 4\}$
   - Dodecahedron: $\{5, 3\}$
   - Icosahedron: $\{3, 5\}$

8. In the same order as the photo above, we have that
   - Tetrahedron: $4 - 6 + 4 = 2$
   - Cube: $8 - 12 + 6 = 2$
   - Octahedron: $6 - 12 + 8 = 2$

- Dodecahedron: $20 - 30 + 12 = 2$
- Icosahedron: $12 - 30 + 20 = 2$

Problem Set Solutions

1. From the lessons, we know 3 different methods: the line segment between any two points is contained within the polygon/polyhedron. The interior angles cannot exceed 180°. The half plane created at any edge contains the whole polygon/polyhedron.
2. From left to right: Convex, Concave, Concave, Concave, Convex
3. Starting on the top, going left to right we have: $k = \frac{11}{12}, \frac{3}{5}, \frac{5}{6}, \frac{2}{5}$
4. This person is incorrect! We know because we have the relationship that all platonic solids follow; for any given Schläfli symbol of a platonic solid,
$$\frac{1}{q} + \frac{1}{p} > \frac{1}{2}$$
So, using the symbol given to us, $\{p, q\} = \{4, 5\}$, we have that
$$\frac{1}{4} + \frac{1}{5} = \frac{9}{20} < \frac{1}{2}$$
So it is not a platonic solid.
5. Euler's characteristic is defined as $\chi = V - E + F$. Going from left to right we have $\chi = 2, 2, 2, 2, 2, 3$.
6.
$$\begin{aligned} (V,E,F) &= (6,12,7) \implies \chi = 1 & (V,E,F) &= (12,24,12) \implies \chi = 0 \\ (V,E,F) &= (20,30,12) \implies \chi = 2 & (V,E,F) &= (4,6,4) \implies \chi = 2 \\ (V,E,F) &= (6,18,12) \implies \chi = 0 & (V,E,F) &= (14,24,12) \implies \chi = 2 \end{aligned}$$
7. Since we have 5 vertices in a pentagram, we have that $p = 5$. When we draw the pentagram, we skip over one vertex every time we connect two vertices, so therefore $q - 1 = 1 \implies q = 2$. Thus, $\{\frac{5}{2}\}$

8.